# Dylan's PHYS211 Notes

# Week 1

**Lecture 1:** *Syllabus Day*

### Vector review

$$v, |\vec{v}| \text{ Magnitude}$$

$$\theta \text{ above } \hat{i}, \text{ Direction}$$

The primary method to add or subtract

$$\vec{A} + \vec{B} = \vec{C} = (\vec{A}_i + \vec{B}_i) + (\vec{A}_j + \vec{B}_j)$$

$$\vec{C}_i = \vec{A}_i + \vec{B}_i$$

$$\vec{C}_j = \vec{A}_j + \vec{B}_j$$

Multiplying vectors

Scalar / Dot Product

$$\vec{A} = c\hat{i} + c\hat{j} + c\hat{k}$$

$$\vec{B} = b\hat{i} + b\hat{j} + b\hat{k}$$

$$\vec{A} \cdot \vec{B} = (\vec{A}_{\hat{i}} + \vec{B}_{\hat{i}}) + (\vec{A}_{\hat{j}} + \vec{B}_{\hat{j}}) + (\vec{A}_{\hat{k}} + \vec{B}_{\hat{k}})$$

Cross Product

$$\vec{A} = c\hat{i} + c\hat{j} + c\hat{k}$$

$$\vec{B} = b\hat{i} + b\hat{j} + b\hat{k}$$

$$\vec{A} \times \vec{B} = A_j B_k - A_z B_j \hat{i} + A_k B_i - A_i B_k \hat{j} + A_i B_j - A_j B_i \hat{k}$$

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**Lecture 2:** *Basic Kinematics*

$$\vec{r}(t) = ct\hat{i} + (ct - gt^2/2)\hat{j} + 0\hat{k}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{dr_x}{dt}\hat{i} + \frac{dr_y}{dt}\hat{j} + 0$$

$$c\hat{i} + (c - gt)\hat{j}$$

$$\vec{a} = (-g)\hat{j}$$

### Kinematics Review

Figure 1: Display of vectors

Position : $\vec{r}(t)$
Length / distance : $r = |\vec{r}(t)|$
Displacement : $\Delta\vec{r}$
Velocity :

$$\vec{v}_{avg} = \frac{\vec{r}(t+\delta t) - \vec{r}(t)}{\Delta t}$$

Instantaneous Velocity :

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \frac{d}{dt}\vec{r}(t)$$

Acceleration :

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \frac{d}{dt^2}\vec{r}(t)$$

**Example problem :** Find velocity (Accelerations) from position (velocity)
Example : Suppose that the position of a particle if given by $\vec{r} = A(e^{\alpha t}\hat{\imath} + e^{-\alpha t}\hat{\jmath})$, where $A$ and $a\alpha$ are constants. Find the velocity and sketch the trajectory.

$$\vec{r} = A(e^{\alpha t}\hat{\imath} + e^{-\alpha t}\hat{\jmath})$$

Velocity would follow from a simple derivative :

$$\frac{d}{dt}\vec{r} = A(\alpha e^{\alpha t}\hat{\imath} + -\alpha e^{\alpha t}\hat{\jmath})$$

We can factor out :

$$\vec{v} = A\alpha(e^{\alpha t}\hat{\imath} - \alpha e^{\alpha t}\hat{\jmath})$$

we can sketch this graph by just looking at the behaviors of the graph at it's extremes.

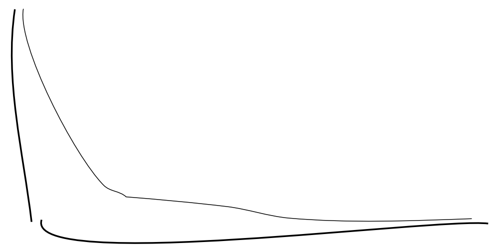

Figure 2: Sample decay graph

A table-tennis ball is released near the surface of the moon with initial velocity $\vec{v}_0 = (0, 5, 3)$ m/s . It accelerates downward with acceleration $\vec{a} = (0, 0, -1.6)$ m/s$^2$. Find its velocity at $t = 5$s

This is a simple problem solved with :

$$\vec{v}(t) = \int \vec{a}(t)\, dt, \int_{t_0}^{t_f} \frac{d\vec{v}}{dt}\, dt = \vec{v}_f - \vec{v}_0$$

$$\int_{t_0}^{t_f} \vec{a}(t)\, dt = \int_{t_0}^{t_f} (0, 0, a_z)\, dt = (0, 0, a_z)t\Big|_{t_0}^{t_f}$$

$$(0, 0, a_z(t_f - t_i)) = (v_{f_x} - v_{0x}, v_{fy} - v_{0y}, v_{fz} - v_{0z})$$

So testing each of the unit vectors :

$$\hat{i} : v_{fx} - v_{0x} \to v_{fz} = 0 \text{ m/s}$$

$$\hat{j} : v_{fy} - v_{0y} \to v_{fy} = 5 \text{ m/s}$$

$$\hat{k} : a_z(t_f - t_i) = v_{fz} - v_{0z} \to v_{fz} = v_{0z} + a_z(t_f - t_i)$$

$$= 3 \text{ m/s } + (-1.6 \text{ m/s}^2)(5s - 0s)$$

$$= 3 \text{ m/s } - 8 \text{ m/s } = -5 \text{ m/s}$$

**Final problem**

A particle is at origin (0,0) when $t = 0$. The velocity of a particle is described by the function $\vec{v} = (3t\hat{i} + 2\hat{j})$ m/s . Find the

Acceleration of the particle

The posotion of the particle at $t = 2$ s

The acceleration is very straightforward :

$$\frac{d}{dt}\vec{v} = (3\hat{\imath} + 0\hat{\jmath})$$

The position of the particle relies on integrating the velocity function then applying the starting point :

$$\vec{d} = \int_0^2 3t\hat{\imath} + 2\hat{\jmath}\,dt$$

$$\vec{d} = \frac{3}{2}t^2\hat{\imath} + 2t\hat{\jmath}|_0^2$$

$$\vec{d} = 6\hat{\imath} + 4\hat{\jmath} + C$$

Since $C$ is the origin, $C = (0\hat{\imath} + 0\hat{\jmath})$ This means our final position vector is :

$$\vec{r} = 6\hat{\imath} + 4\hat{\jmath}$$

## Week 2

**Lecture 3:** *Polar Coordinates*

The Cartesian coordinates of a particle are (2m,1m).
What are its polar coordinates $(r, \theta)$? Write the position vector of the particle in the cartesian coordinate system and in the polar coordinate system respectively.

$$\vec{r} = (2\text{ m}, 1\text{ m } = 2\text{ m }\hat{\imath} + 1\text{m }\hat{\jmath})$$

$$r = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{1}{2}\right)$$

$$r = \sqrt{5}, \theta = 0.464$$

$$\vec{r} = r\hat{r}$$

You don't need to specify $\hat{r}$'s direction, it's a unit vector that goes along the vector being described.

Converting between $\hat{\imath}, \hat{\jmath}$ and $\hat{r}, \hat{\theta}$

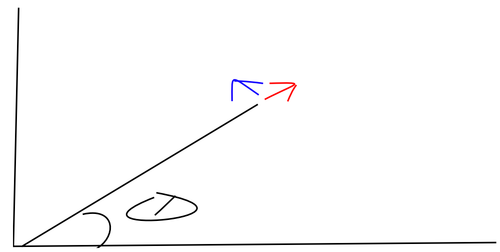

Figure 3: PolarReference

$$\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$$

$$\hat{\theta} = -\sin\theta\hat{i} + \cos\theta\hat{j}$$

$$\hat{i} = \frac{x}{r}\hat{r} - \frac{y}{\hat{y}}$$

$$\vec{r} = r\hat{r} = r\cos\theta\hat{i} + r\sin\theta\hat{j}$$

$$\vec{v} = \frac{d}{dt}\vec{r} = \dot{\vec{r}}$$

$$\vec{a} = \frac{d}{dt}d\vec{v} = \dot{\vec{v}} = \frac{d^2}{dt^2}\dot{\vec{r}} = \ddot{\vec{r}}$$

In cartesian coordinates

$$\vec{v} = (\dot{x}, \dot{y}, \dot{z}) = \dot{x}\hat{i} + \dot{y}\hat{i} + \dot{z}\hat{k}$$

In polar coordiantes...What are $\dot{r}$ and $\dot{\theta}$?

The time rate of change of the coorindates $r\&\theta$

$$\vec{v} = \dot{\vec{r}} = \frac{d}{dt}(r\hat{r}) \to \dot{r}\hat{r} + r\frac{d}{dt}\hat{r}$$

$$\frac{d}{dt}\hat{r} = \frac{d}{dt}(\cos\theta\hat{i} + \sin\theta\hat{j}) = \dot{\theta}(-\sin\theta)\hat{i} + \dot{\theta}\cos\theta\hat{j}$$

$$= \dot{\theta}(-\sin\theta\hat{i} + \cos\theta\hat{j}) = \dot{\theta}\hat{o}$$

$$\frac{d}{dt}\hat{r} = \dot{\theta}\hat{\theta} = \frac{d}{dt}\vec{r} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

Remember that $\dot{\theta} = \omega$

$\dot{r}$ describes the change in vector length $\dot{\theta}$ describes the change in vector direction.

$$\vec{v} = \vec{r}\frac{d}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

The velocity of a particle in circular motion can easily be derived by

$$v = R\omega$$

Which makes conceptual sense as the rate of cycles times radius of the cycle would intuitively produce a speed.

Position and velocity in polar coordinates.

$$\vec{r} = r\hat{r}$$

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

What about acceleration?

$$\vec{a} = \frac{d}{dt}\vec{v} = \frac{d}{dt}(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta})$$

$$\vec{a} = \ddot{r}\vec{r} + \dot{r}\dot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$$

$$= (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \rightarrow \text{ Acceleration } \parallel \text{ to}$$

$$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} \rightarrow \text{ Acceleration } \perp \text{ to } \vec{r}$$

If $r$ was constant, $\vec{a} = -r\dot{\theta}^2\hat{r} + r\ddot{\theta}\hat{\theta}$

$$\ddot{r} \rightarrow \text{ Change in radial speed}$$

$$2\dot{r}\dot{\theta} \rightarrow \text{ Coriolis acceleration}$$

Table 1: Cartesian vs Table Summary

| Kinematics | Cartesian | Polar/Cylindrical |
|---|---|---|
| Position | $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ | $\hat{r} + r\hat{r} + 0\hat{\theta} + z\hat{k}$ |
| Velocity | $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$ | $\dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{k}$ |
| Acceleration | $\vec{a} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$ | $(\ddot{r} - r\theta^2)\hat{r} + (2\dot{r}\theta + r\theta)\theta$ |

A particle moves along a circular path of radius $R$. Its angular position is given by

$\theta = \alpha t^2/2$. What is the acceleration of the particle at time $t$?

$$\theta = \alpha \frac{t^2}{2}$$

$$\dot{\theta} = \alpha t$$

$$\ddot{\theta} = \alpha$$

$$(\ddot{r} - r\theta^2)\hat{r} + (2\dot{r}\theta + r\theta)\theta$$

Since we know that the radius doesn't change we can drop many terms :

$$\alpha = -r\left(\alpha \frac{t^2}{2}\right)\hat{r} + \left(r\alpha \frac{t^2}{2}\right)\hat{\theta}$$

A mass $m$ on the end of a string of length $R$ whirls in free space in a horizontal plane, with constant speed $v$. The force on $m$ is :

$$-\frac{mv^2}{R}\hat{r}$$

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**Lecture 4:** *Newton's Laws*

A newton's Third Law pair only happens across different objects, it occurs when something pushes back to resist the inertia of of something else.

# Week 3

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**Lecture 5:** *Just in class prob- lems.*

We literally did not do much at all during this day. We primarily just covered the problems in class from the previous day and everything was simple.

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**Lecture 6:** *Friction*

Friction and Spring forces will be the primary topic for the next week.

When a block is on an incline at rest, it experiences 3 distinct forces, gravity and the normal forces. But also notably static friction. I was under the impression that static friction is not a force that can exist without motion, this is wrong. Friction opposes forces not movement.

Scenario 1 : $\mu_s$ static friction needs to be overpowered to cause motion

Scenario 2 : $F_s = N\mu_s$ This is the minimum amount of force required to break static friction

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**Lecture 7:** *Gravitation*

Mass $m$ is whirled at instantaneous speed $v$ on the end of a string of length $R$ The motion is in a vertical plane near the surface of the Earth. The string makes instantaneous angle $\theta$ with the horizontal. Find the magnitude of the tensions, $T$ in the string and the tangential acceleration at this instant.

We know the typical tension on a mass restricted by a string is

$$T = \frac{mv^2}{r}$$

We know the tension on the string decreases when the object is at the top of it's arc, and we know that the tension on the string is maximized when it's on the bottom of it's motion.

# Week 4

**Lecture 8:** *Practice for Exam*

Consider a vibrating star, whose frequency $\nu$ depends on its radius $R$, mass density $\rho$, and newton's gravitational constant $G$. How does $\nu$ depend on $R$, $\rho$, and $G$

$$v = T^{-1}$$

$$R = L$$

$$G = M^{-1}L^3T^{-2}$$

$$\rho = ML^{-3}$$

$$v = \sqrt{G\rho}$$

A person throw a ball with speed $v$ from the edge of a cliff of height $h$. What is the maximum horizontal distance he can throw?

$$v_x = v_0 \cos\theta$$

$$v_y = v_0 \sin\theta$$

$$x_x = tv_0 \cos\theta$$

$$x_y =$$

Time for free fall :

$$t = \sqrt{\frac{2h}{g}}$$

$$x_{\text{max}} = vt = v\sqrt{\frac{2h}{g}}$$

A ball is swinging around on a thether hanging down at an angle from a pole. The ball has mass $m$ the rope has length $L$ and the ball moves in a horizontal circle at speed $v_i$. Find :

- the tension in the rope.

$$T_y = T\cos\alpha$$

$$T_x = T\sin\alpha$$

$$T_y = mg$$

$$T = \frac{mg}{\cos\alpha}$$

- The angle $\alpha$ at which the rope hangs down