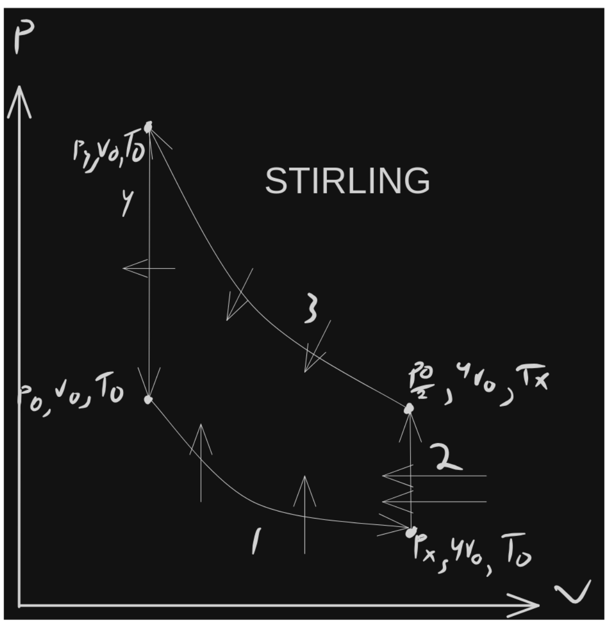
Dylan Sheehan's 4th Ch302 Homework

1.

The Stirling engine operates on the following cycel beginning at $P_0, V_0,$ and T_0
I. Isothermal expansion to $4V_0$
II. Constant volume heating to $\frac{P_0}{2}$
III. Isothermal compression to V_0
IV. Constant volume cooling back to T_0
Sketch the cycle on a $P - V$ diagram. Calculate the change in T for each leg in terms of T_0, the net work done by the cycle the heat absorbed by the cycle, and determine the efficiency of the engine. Compare this efficiency to the Carnot cycle efficiency operating between the highest and lowest temperatures of this cycle. Treat the gas as 1 mole of ideal gas, with a value of $C_v = 3R$
PV=NR



Now we must find $q + w$ for each step! And find P_x, P_y, T_x.

$$P_x = \frac{P_0}{4}$$

$$T_x = 2T_0$$

$$P_y = 2P_0$$

$$w_1 = - \Bigg| q_1 = +$$

$$w_2 = 0 \Bigg| q_2 = +$$

$$w_3 = + \,\Big|\, q_3 = -$$

$$w_4 = 0 \,\Big|\, q_4 = -$$

Step 1 :

$$(P_0, V_0, T_0) \rightarrow \left(\frac{P_0}{4}, 4V_0, T_0\right)$$

$$w_1 = \underbrace{-RT_0 \ln\left(\frac{4V_0}{V_0}\right)}_{\text{Isotherm work}} = 2(0.693)RT_0 = 1.386RT_0$$

$$q_1 = \underbrace{-w_1}_{heat} = 1.386RT_0$$

Step 2:

$$\left(\frac{P_0}{4}, 4V_0, T_0\right) \rightarrow \left(\frac{P_0}{2}, 4V_0, 2T_0\right)$$

$$w_2 = 0$$

$$q_2 = c_v \Delta T = 3R(2T_0) = 6RT_0$$

Step 3:

$$\left(\frac{P_0}{2}, 4V_0, 2T_0\right) \rightarrow (2P_0, V_0, 2T_0)$$

$$w_3 = -R(2T_0)\ln\left(\frac{V_0}{4V_0}\right) = 2.772RT_0$$

$$q_3 = -w_3 = -2.772RT_0$$

Step 4:

$$(2P_0, V_0, 2T_0) \rightarrow (P_0, V_0, T_0)$$

$$w_4 = 0$$

$$q_4 = c_v \Delta T = (3R)(T_0 - 2T_0) = -3RT_0$$

$$U_{cycle} = q_{cycle} + w_{cycle} = 0$$

$$(1.386RT_0 + 3RT_0 - 2.772RT_0 - 3RT_0) + (1.386RT_0 + 2.772RT_0)$$

$$(-1.386RT_0) + (1.386RT_0) = 0$$

$$q_{net} = - \,\Big|\, w_{net} = +$$

We are adding work to engine to extract heat in this case. Which makes it a refrigerant

method.
Efficiency for a heat engine :

$$\frac{\text{Net work}}{\text{Heat in}}$$

$$\frac{|\text{sum of all work}|}{+q}$$

$$\frac{1.386RT_0}{4.386RT_0}$$

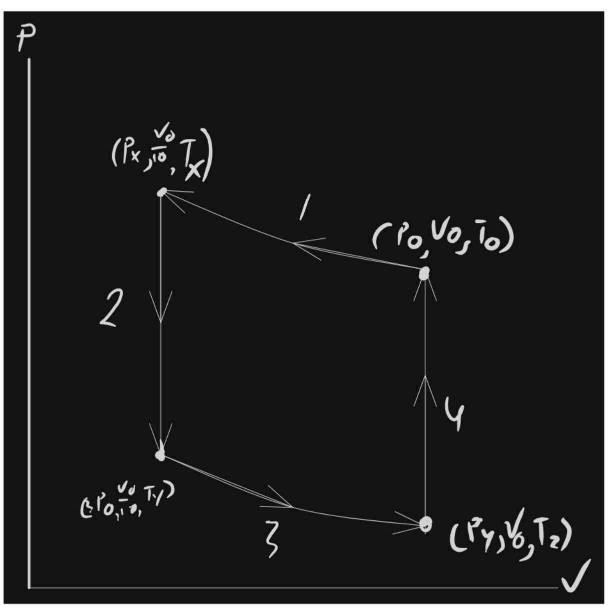
$$n = \text{ efficiency} = \frac{1.3686RT_0}{4.386RT_0} = 31.6\,\%$$

This value doesn't make much sense because it's a refrigerant cycle.
Carnot efficiency $\frac{T_{high}-T_{low}}{T_{high}}$

$$\frac{T_{high}-T_{low}}{T_{high}} = \frac{2T_0 - T_0}{2T_0} = \frac{1}{2} = 50\,\% \text{ efficient}$$

2.

The Otto engine operates on the following cycle beginning at P_0, V_0, T_0:
I. Adiabatic compression to $\frac{V_0}{10}$
II. Constant volume cooling to $3P_0$
III. Adiabatic expansion to V_0
IV. Constant volume heating back to T_0
Sketch the cycle on a $P-V$ diagram. Calculate the change in T for each leg in terms of T_0, the net work done by the cycle, the heat absorbed by the cycle, and determine the efficiency of the engine. Treat the gas as 1 mole of ideal gas, with a value of $C_v = 2.5R$



$$PVc_v = 1$$

$$c_v = 2.5R$$

$$\underbrace{c_p = c_v + R}_{\text{ideal gas}}$$

$$c_p = 3.5R$$

$$\gamma = \frac{c_p}{c_v} = \frac{3.5R}{2.5R} = 1.4$$

$$w = c_v \Delta T$$

$$w_1 = - \Big| q_1 = 0$$

$$w_2 = 0 \Big| q_2 = +$$

$$w_3 = + \Big| q_3 = 0$$

$$w_4 = 0 \Big| q_4 = -$$

Step 1 :

$$(P_0, V_0, T_0) \to \left(P_x, \frac{V_0}{10}, T_x\right)$$

Calculating P_x :

$$P_0 V_0^{\gamma} = 1$$

$$\frac{P_0 V_0^{\gamma}}{T_0} = \frac{P_x \frac{V_0^{\gamma}}{10}}{T_x}$$

$$P_x \left(\frac{V_0}{10}\right)^{1.4} = P_0 V_0^{1.4}$$

$$P_x = 10^{1.4} P_0$$

$$\frac{10^{1.4} \cancel{P_0} \frac{V_0^{\gamma}}{10}}{T_x} = \frac{\cancel{P_0} V_0^{\gamma}}{T_0}$$

$$\frac{1}{T_x} = \frac{10}{10^{1.4} T_0}$$

$$T_x = 10^{0.4} T_0$$

$$w = c_v \Delta T$$

$$w = 2.5R \times (10^{0.4} T_0 - T_0) = 3.75 T_0$$

$$q = 0$$

Step 2 :

$$\left(10^{1.4}P_0, \frac{V_0}{10}, 10^{0.4}T_0\right) \rightarrow \left(3P_0, \frac{V_0}{10}, T_y\right)$$

Finding T_y

$$\frac{10^{1.4}P_0\left(\frac{V}{10}\right)^{\gamma}}{10^{0.4}T_0} = \frac{3P_0\left(\frac{V}{10}\right)^{\gamma}}{T_y}$$

$$\frac{10}{T_0} = \frac{3}{T_y}$$

$$T_y = \frac{3T_0}{10}$$

As the heating is isochoric, the work is 0.

$$w = 0$$

$$q = c_v \Delta T = 2.5R \times (0.3T_0 - 2.5T_0) = -5.5RT_0$$

Step 3:

$$\left(3P_0, \frac{V_0}{10}, \frac{3T_0}{10}\right) \rightarrow (P_y, V_0, T_z)$$

Finding pressure :

$$3P_0\left(\frac{V_0}{10}\right)^{\gamma} = P_y V_0^{\gamma}$$

$$\frac{3P_0}{10^{\gamma}} = P_y$$

$$\frac{3P_0}{10^{1.4}} = P_y$$

Finding Temperature :

$$\frac{\cancel{3P_0}\left(\frac{\cancel{V_0}}{10}\right)^{\gamma}}{\frac{3T_0}{\cancel{10}}} = \frac{\frac{\cancel{3P_0}}{\cancel{10}}\cancel{V_0^{\gamma}}}{T_z}$$

$$T_z = 0.12T_0$$

$$w = C_v \Delta T = 2.5R(0.12T_0 - 0.3T_0) = -0.45RT_0$$

$$q = 0$$

Step 4:

$$\left(\frac{3P_0}{10^{\gamma}}, V_0, 0.12T_0\right) \rightarrow (P_0, V_0, T_0)$$

There is no work here as this is an isochoric change.

$$w = 0$$

$$q = C_v \Delta T = 2.5R(T_0 - 0.12T_0) = 2.2RT$$

Comparing work to carnot efficiency.

$$\text{Carnot} : \frac{2.5T_0 - 0.92T_0}{2.5T_0} = 95\,\%$$

$$\text{Efficiency} : \frac{3.75T_0 - 0.45T_0}{2.2RT_0} = 150\,\%$$

These values make no sense because we're working with a refrigeration cycle.